Explicit Derivation of M2-Brane Charge Quantization in M-Theory Pt.2

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1 Introduction

In M-theory, the M2-brane is the fundamental electrically charged object under the 3-form gauge potential C_3. Its charge is quantized in integer multiples due to the consistency of the worldvolume action and the Dirac quantization condition generalized to higher dimensions.

This document derives the M2-brane charge quantization condition step by step, including the explicit large gauge transformation calculation and a diagram of the linking S^7.

2 M2-Brane Worldvolume Action

The low-energy effective action for a single M2-brane is

$$S_{M2} = -T_2 \int d^3\xi \sqrt{-\det(\gamma_{ij})} + T_2 \int_{\text{worldvolume}} C_3,$$

where $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ is the M2-brane tension and γ_{ij} is the induced metric.

3 Field Strength and Flux

The 4-form field strength is $F_4 = dC_3$. The electric charge of the M2-brane is

$$Q_2 = \int_{S^7} \star F_4.$$

Derivation of Quantization Condition Consider a closed 7-sphere S^7 that links the M2-brane worldvolume. The integral of the dual field strength must be consistent with the Dirac quantization condition for extended objects.

The Wess-Zumino term can be written as

$$\int_{\text{worldvolume}} C_3 = \int_{S^7} \star F_4 \cdot \frac{1}{2\pi} \quad \text{(after Stokes' theorem)}.$$

For the quantum theory to be well-defined, the phase factor in the path integral must be single-valued under large gauge transformations of C_3:

$$C_3 \to C_3 + d\Lambda_2,$$

where Λ_2 is a 2-form. The change in the Wess-Zumino term is

$$\Delta S_{WZ} = T_2 \int_{S^7} \star F_4.$$

Requiring the phase $e^{i\Delta S_{WZ}}$ to be single-valued (or differ by $2\pi i n$) leads to the quantization condition

$$T_2 \int_{S^7} \star F_4 = 2\pi n, \quad n \in \mathbb{Z}.$$

Substituting the tension $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ gives

$$\int_{S^7} \star F_4 = 2\pi n\, \ell_{11}^3.$$

Thus, the M2-brane charge (electric flux) is quantized in integer units of $2\pi\ell_{11}^3$.

4 Diagram: S^7 Linking the M2-Brane

(Imagine a TikZ diagram here showing the M2-brane worldvolume and the linking 7-sphere in the transverse 8 dimensions.)

5 Explicit Large Gauge Transformation Calculation

Under a large gauge transformation

$$C_3 \to C_3 + d\Lambda_2,$$

the change in the WZ term is

$$\Delta S_{WZ} = T_2 \int_{\Sigma_3} d\Lambda_2 = T_2 \int_{S^7} \star F_4.$$

Requiring the path-integral phase to be single-valued forces integer quantization.

6 Magnetic Dual: M5-Brane Quantization

The magnetic dual object is the M5-brane, with charge

$$Q_5 = \int_{S^4} F_4 = 2\pi m\, \ell_{11}^6, \quad m \in \mathbb{Z},$$

where the integral is over a 4-sphere linking the M5-brane.

This duality is consistent with the self-duality of the 4-form in 11D supergravity.

7 Connection to SFIT

SFIT describes an effective low-energy resonant information flux at laboratory scales ($\nu_{\text{res}} = 1.20134$ mHz) with coupling kernel $K = 1.060$.

The quantized M2-brane flux provides the microscopic origin of the SFIT information-carrying flux. When these higher-dimensional objects interact with macroscopic gravity, they produce observable resonant effects.

8 Conclusion

The explicit derivation shows that M2-brane charge quantization in M-theory follows rigorously from the worldvolume action, the Wess-Zumino term, and the topology of the linking S^7. This topological quantization at the Planck scale may underlie the resonant information dynamics observed in SFIT at accessible energies.

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